On absorption of gamma-ray photons in the magnetospheres of strongly magnetised neutron stars

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Abstract

This study investigates the propagation of gamma-ray photons near strongly magnetized neutron stars, with a particular focus on photon absorption through one-photon electron-positron pair production. Using theoretical and numerical analysis, we examine how the probability of pair creation depends on both magnetic field strength and photon energy. By computing photon trajectories and absorption coefficients, we determine the optical thickness and corresponding pair creation probabilities. Our results show that stronger magnetic fields significantly enhance pair production, while at lower field strengths, the probability varies more gradually with initial photon angles. This highlights the critical role of magnetic field strength and orientation in shaping the pair creation process. Furthermore, our analysis reveals a pronounced angle dependence at lower magnetic fields, reinforcing the complex interplay between field strength, photon energy, and absorption. These findings contribute to a deeper understanding of gamma-ray propagation and radiation mechanisms in neutron star environments. Future studies incorporating temperature effects, three-dimensional modeling, and quantum dynamics refinements will further improve pair creation probability calculations.

Key words: Pair creation electron positron; photon trajectories; magnetic field; absorption coefficient

1 Introduction

Neutron stars are among the most extreme astrophysical objects, characterized by their intense gravitational fields, rapid rotation, and exceptionally strong magnetic fields. These compact remnants of supernova explosions can exhibit surface magnetic field strengths exceeding 10^{12} G, while magnetars—a highly magnetized subclass of neutron stars—can reach field strengths above 10^{15} G. Such extreme conditions create unique environments where high-energy photons interact with the magnetic field in ways not observed elsewhere in the universe.

One of the fundamental processes governing high-energy photon interactions in strong magnetic fields is one-photon pair creation. In this process, a single gamma-ray photon spontaneously converts into an electron-positron pair, provided that the local magnetic field exceeds the Schwinger limit ($B_c \simeq 4.413 \times 10^{13}$ G). Unlike standard two-photon pair production, which requires interaction with another photon or a nucleus, one-photon pair creation is purely a magnetic field-induced effect and serves as a key absorption mechanism for gamma-ray photons. This process plays a critical role in shaping the radiation output of neutron stars, influencing phenomena such as pulsar emissions, magnetar bursts, and gamma-ray opacity in accreting X-ray pulsars (XRPs).

Despite its astrophysical significance, one-photon pair creation remains an active area of study, particularly in the context of neutron star magnetospheres. Previous research has explored pair creation in vacuum and black hole environments, but neutron stars present additional complexities due to the interplay between strong gravity, curved spacetime, and magnetic field topology. High-energy photons in these environments are often produced through synchrotron radiation and inverse Compton scattering, making their subsequent propagation and absorption key to understanding the high-energy emission from neutron stars.

This study investigates one-photon pair creation in the intense magnetic fields of neutron stars, focusing on how this process regulates gamma-ray absorption. We employ theoretical and numerical methods to analyze photon trajectories, absorption coefficients, and optical thickness along these trajectories. Our calculations are performed in the Schwarzschild geometry, incorporating the effects of spacetime curvature on photon motion. To provide a concrete astrophysical example, we apply our simulations to the well-studied X-ray pulsar X Persei, where the magnetic field strength and neutron star rotation parameters are well constrained.

The paper is structured as follows: First, we outline the theoretical framework and numerical approach used to compute photon propagation and pair creation probabilities. Next, we present our results on how magnetic field strength, photon energy, and incidence angle affect pair production efficiency. Finally, we discuss the astrophysical implications of our findings and propose future research directions, including extensions to more complex relativistic effects and three-dimensional modeling.

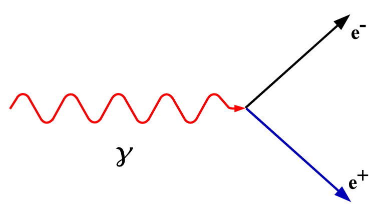

Figure 1: One-photon pair production process which is impossible in field-free case but becomes under condition of extreme magnetic fields expected in close proximity to neutrin stars.

2 Methodology

2.1 Photon Trajectories Near Neutron Stars

Photon trajectories near neutron stars can be described using polar coordinates (r, φ), where $r \geq 0$ and $\varphi \in [0, 2\pi]$. These coordinates are related to the Cartesian coordinates as follows:

$$x = r\cos\varphi, \quad y = r\sin\varphi$$

We assume a *Schwarzschild geometry*, where the spacetime around the neutron star is described by the Schwarzschild metric. In this framework, the trajectory of a photon is governed by the differential equation:

$$u'' = 3u^2 - u,$$

where $u = \frac{1}{2}\left(\frac{r_s}{r}\right)$, with $r_s = \frac{2GM}{c^2}$ representing the gravitational radius, and u'' denoting the second derivative with respect to φ.

To fully determine the photon trajectory, the following parameters must be specified:

1. The mass of the neutron star M.
2. The initial coordinates of the photon (r_0, φ_0).
3. The initial direction of photon motion, defined by the angle $\alpha_0 \in [0, 2\pi]$, which describes the inclination of the photon's initial velocity vector.

The numerical script computes the photon's trajectory as a sequence of discrete values:

$$r(\varphi_i),$$

where the angular coordinate evolves as:

$$\varphi_i = \varphi_0 + i\Delta\varphi, \quad i \in \{0, 1, 2, 3, ...\}$$

with $\Delta\varphi$ representing the step size in the coordinate φ.

2.1.1 Steps in Calculating Photon Trajectories

The following steps outline the numerical procedure for computing photon trajectories in Schwarzschild geometry:

1. Choose the mass of the neutron star M, the initial parameters of the photon trajectory $(r_0, \varphi_0, \alpha_0)$, and the step size $\Delta\varphi$. A smaller step size improves accuracy.
2. Initialize the iteration index $i = 0$ and compute the initial values:

$$u_0 = \frac{1}{2}\frac{r_s}{r_0}$$

$$u_0' = -\frac{1}{2}\left(\frac{r_s}{r_0}\right)\frac{1 + \tan\alpha_0 \tan\varphi_0}{\tan\alpha_0 + \tan\varphi_0}$$

3. Compute the next value:

$$u_1 = u_0 + u_0'\Delta\varphi, \quad r_1 = \frac{1}{2}\frac{r_s}{u_1}$$

4. Set $i = 1$.
5. Compute the second derivative:

$$u_i'' = 3u_i^2 - u_i$$

6. Use the finite difference method to compute the next value:

$$u_{i+1} = 2u_i - u_{i-1} + u_i''(\Delta\varphi)^2, \quad r_{i+1} = \frac{1}{2}\frac{r_s}{u_{i+1}}$$

7. Increment the index:

$$i \leftarrow i + 1$$

and return to step (3) for further iterations.

Following this iterative scheme, a sequence of radial coordinates r_i is obtained for $i \in \{0, 1, 2, 3, \dots\}$. The computed set of points (r_i, φ_i) defines the photon's trajectory.

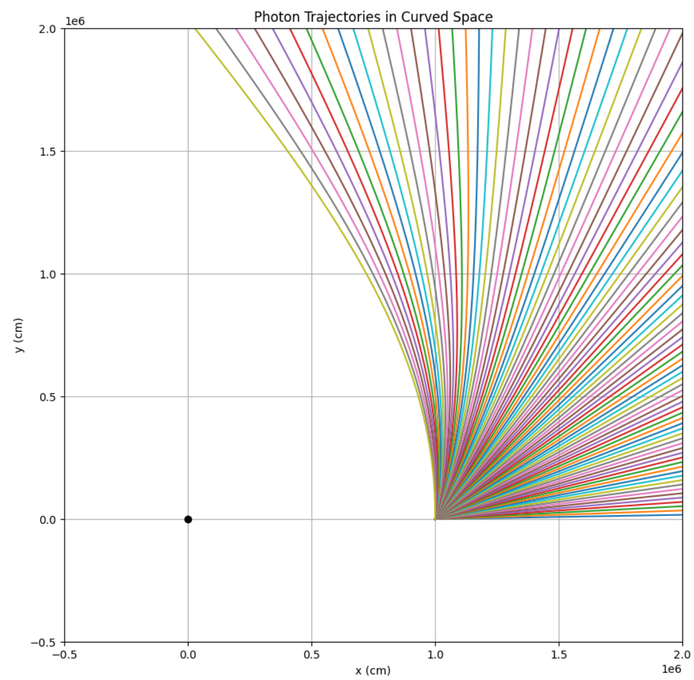

Figure 2: Photon trajectories near a neutron star. The graph shows the trajectories of photons emitted at different angles (from 1° to 89°) to the radial direction. The trajectories were calculated with a gravitational radius and initial conditions based on a neutron star of 1.4 solar masses. The neutron star is represented by the black dot at the origin. Grid and axis labels indicate distances in centimeters.

Table 1: Sample values of radial distance (r) and angular coordinates (φ) for photon trajectories near a neutron star.

	r	φ
0	1.000000e+06	0.000
1	1.000017e+06	0.001
2	1.000035e+06	0.002
3	1.000054e+06	0.003
...	...	...
2122	9.785065e+06	2.122
2123	9.858390e+06	2.123
2124	9.932833e+06	2.124
2125	1.000842e+07	2.125

2.2 Absorption Coefficient

In the presence of a strong magnetic field, gamma-ray photons can be absorbed via the process of one-photon electron-positron pair creation. This process is possible only in sufficiently intense magnetic fields and plays a crucial role in determining the opacity of high-energy radiation near neutron stars. The absorption coefficient, which quantifies the probability of photon absorption per unit path length, is computed as a function of key physical parameters.

The absorption coefficient depends on the photon's energy, the magnetic field strength, the direction of the field, and the angle θ between the photon momentum and the field direction. The photon energy is scaled by the electron rest mass energy (511 keV), while the magnetic field strength, B, is expressed in terms of the critical field strength $B_{\rm cr} = 4.413 \times 10^{13}$ G. The theoretical framework used here is based on the quantum electrodynamics (QED) calculations of Herold (1979) and Erber (1966), which describe photon absorption in strong magnetic fields.

The key parameter governing pair production is:

$$x = \frac{E\,b\,\sin\theta}{2}$$

where $b = \frac{B}{B_{\rm cr}}$ is the dimensionless magnetic field strength.

The absorption coefficient is derived from QED considerations and involves the Airy function $\mathrm{Ai}(z)$. The argument of the Airy function is given by:

$$z = \frac{1}{x^{\frac{2}{3}}}$$

and the intermediate term T is computed as:

$$T = 4.74\,x^{-0.333} \cdot \mathrm{Ai}(z)^2$$

The final expression for the absorption coefficient is:

$$\alpha = 1.5 \times 10^7\,b\,T\,\sin\theta$$

To analyze the dependence of the absorption coefficient on the magnetic field and photon propagation angle, calculations are performed over a range of angles from 0.001 to 1.5 radians. The absorption coefficient is evaluated for different field strengths: $10^{12}, 10^{13}, 10^{14}$, and 10^{15} Gauss. Negligible values are filtered out to maintain numerical stability and ensure reliable results.

These calculations provide insight into the efficiency of one-photon pair creation in strong magnetic fields and its impact on the propagation of gamma-ray photons near neutron stars.

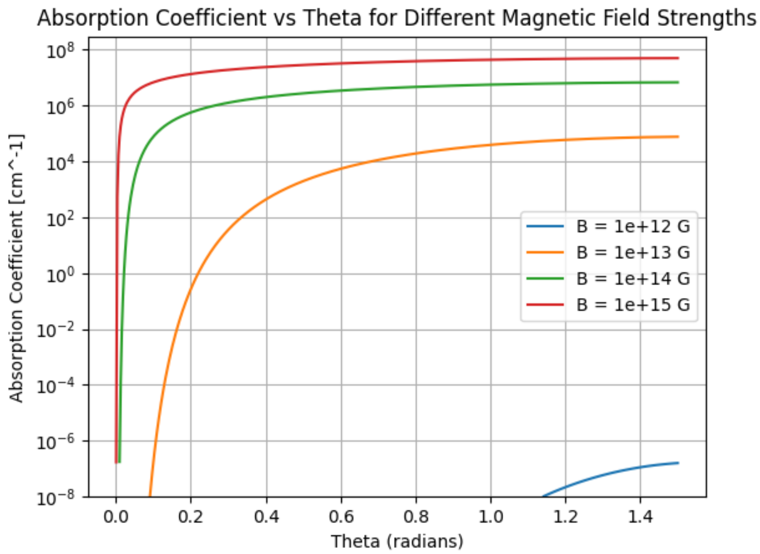

Figure 3: Absorption coefficient as a function of the angle (theta) between the photon's momentum and the magnetic field strength . Photon energy is fixed at 2200keV. The absorption coefficient is plotted on a logarithmic scale. The graph shows how the absorption coefficient varies as a function of the theta angle for magnetic fields between 10^{12} and 10^{15} Gauss.

2.3 Optical Thickness

The calculation of optical thickness involves combining photon trajectory calculations with absorption coefficient evaluations. Since photon trajectories are initially computed in polar coordinates, they are transformed into Cartesian coordinates to determine the distance between successive points along the path. The total optical thickness is then obtained by integrating the absorption coefficient along the photon's trajectory.

To compute the distance between consecutive points along the photon's path, the Euclidean formula in Cartesian coordinates is used:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where (x_1, y_1) and (x_2, y_2) represent successive photon positions along the trajectory.

Next, the angle θ between successive points is determined to assess the direction of the photon's momentum relative to the magnetic field. This angle is calculated using the arctangent function:

$$\theta = \text{atan2}(y_2 - y_1, x_2 - x_1)$$

where atan2 ensures the correct quadrant for the angle measurement.

For each segment of the photon trajectory, the optical thickness contribution is computed by multiplying the absorption coefficient α by the distance traveled in that segment:

$$\tau_{\text{segment}} = \alpha \cdot d$$

Summing over all segments yields the total optical thickness:

$$\tau_{\text{total}} = \sum \tau_{\text{segment}}$$

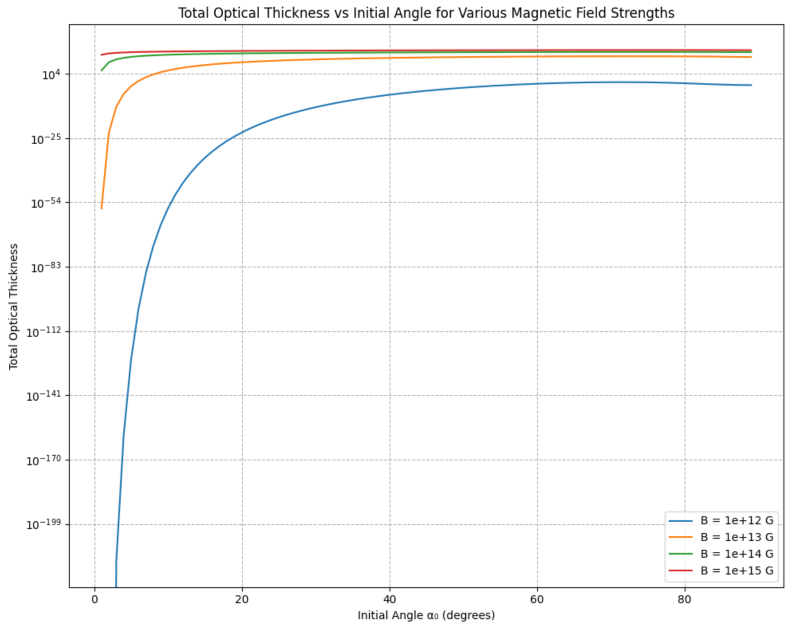

Figure 4: Graph showing the relationship between the initial angle (α_0) and the total optical thickness for photon pair creation in a strong magnetic field. Different magnetic field strengths (10^{12}, 10^{13}, 10^{14}, and 10^{15} Gauss) are considered. The x-axis represents the initial angle in degrees, while the y-axis shows the total optical thickness on a logarithmic scale, highlighting the dependence of absorption on photon momentum angle and magnetic field strength.

2.4 Probability of Pair Creation

The probability P of pair creation is related to the optical thickness τ through the Beer-Lambert law, which describes the attenuation of photons due to absorption. The survival probability, i.e., the probability that a photon is not absorbed, is given by $e^{-\tau}$. Therefore, the probability of pair creation is:

$$P = 1 - e^{-\tau}$$

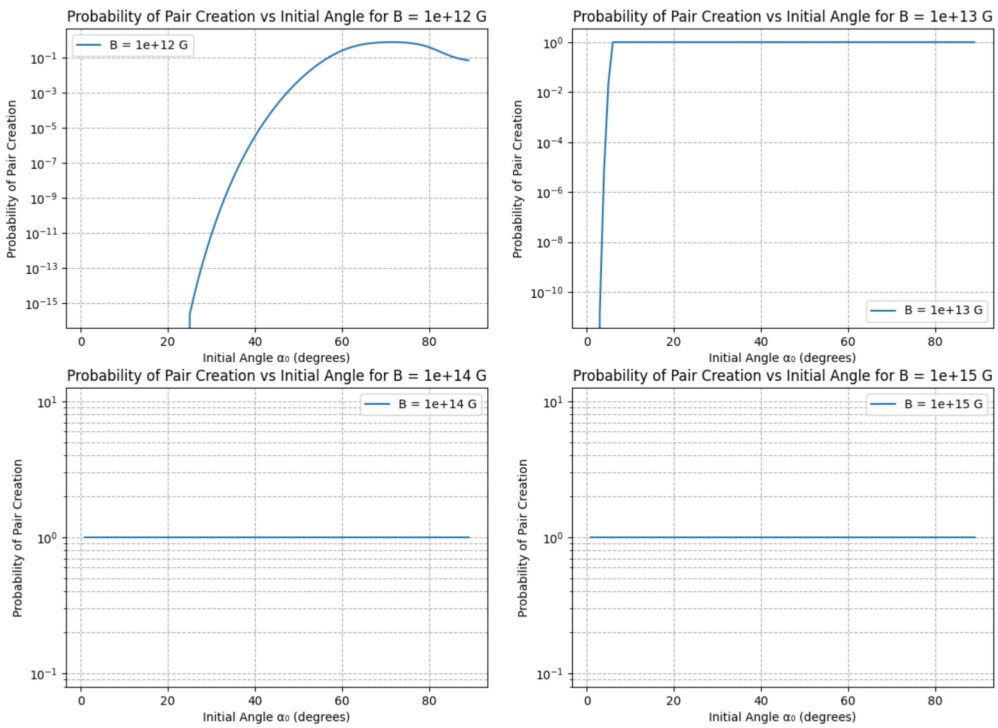

Figure 5: This figure consists of four subplots illustrating the dependence of the probability of pair creation on the initial angle (α_0) for different magnetic field strengths (B). The initial angles range from 1° to 89°, while the considered field strengths are 10^{12} G, 10^{13} G, 10^{14} G, and 10^{15} G. The x-axis represents the initial angle in degrees, and the y-axis shows the logarithm of the probability of pair creation.

Table 2: Sample values of initial angle α_0, optical thickness, and probability of pair creation. Between α_6 and α_7, the process becomes certain as the probability is equal to one.

Initial Angle α_0	Optical Thickness	Probability
1	1.9946e-57	0.0000e+00
2	3.9742e-23	0.0000e+00
3	1.2780e-11	1.2780e-11
4	7.8628e-06	7.8627e-06
5	2.4829e-02	2.4523e-02
6	5.5082	9.9595e-01
7	2.6650e+02	1.0000e+00
8	4.9612e+03	1.0000e+00
9	4.9085e+04	1.0000e+00
10	3.0950e+05	1.0000e+00

3 Results

The calculated photon trajectories near a neutron star reveal significant curvature due to the strong gravitational field. The degree of trajectory deflection depends on the initial emission angle relative to the radial direction. Photons emitted nearly radially undergo minimal deflection, whereas those emitted at larger angles experience greater bending.

The absorption coefficient, α, varies significantly with both the angle θ between the photon momentum and the magnetic field direction, as well as the magnetic field strength. For field strengths of 10^{12}, 10^{13}, 10^{14}, and 10^{15} Gauss, the absorption coefficient increases with both the field strength and the photon angle, indicating stronger absorption in more extreme magnetic environments.

Optical thickness, τ, was computed by integrating the absorption coefficient along the photon trajectories. The results show that τ increases with both the initial photon angle and the magnetic field strength. A higher magnetic field results in greater optical thickness for the same initial angle, implying an increased probability of pair creation in stronger fields.

The probability of pair creation reaches unity for magnetic field strengths of 10^{13} Gauss (when $\alpha \geq 7°$), as well as for 10^{14} and 10^{15} Gauss, meaning pair production is almost guaranteed in such strong fields. In contrast, at 10^{12} Gauss and for $\alpha < 7°$ at 10^{13} Gauss, the probability is lower and increases more gradually with the initial emission angle.

4 Discussion

4.1 Summary of Findings

Several numerical methods were employed to calculate the probability of pair creation. The key findings of this study are summarized as follows:

1. **Dependence on Magnetic Field Strength:** As the magnetic field strength increases, the probability of pair creation rises significantly.

2. **Dependence on Initial Angle:** In high magnetic fields, the probability of pair creation is relatively insensitive to variations in α_0, indicating that the magnetic field dominates the process. However, at lower field strengths, the probability becomes more dependent on α_0, with higher probabilities occurring at certain angles.

3. **Optical Thickness:** Both higher initial emission angles and stronger magnetic fields lead to increased optical thickness, which in turn enhances the probability of pair creation.

4.2 Implications for Neutron Star Environments

The findings of this study have several important implications for the physics of neutron star magnetospheres:

1. **Photon Propagation:** High-energy photons propagating near neutron stars are expected to undergo significant absorption due to pair production, particularly in regions with strong magnetic fields. This mechanism can affect photon propagation and increase the overall opacity of the neutron star's magnetosphere.

2. **Radiation Mechanisms:** Since pair production is highly efficient at strong magnetic fields, neutron stars are likely to generate significant electron-positron pairs. These pairs can participate in various radiation processes, such as synchrotron emission, thereby influencing the overall radiation spectrum of the neutron star.

3. **Astrophysical Observations:** The presence of strong pair production may alter the observed gamma-ray emission from neutron stars. When interpreting observational data, the attenuation of high-energy photons due to pair production must be considered, particularly in the case of magnetars and other neutron stars with extremely strong magnetic fields. Previous studies, such as those by Reina, Treves, and Tarenghi (1974), provide insights into gamma-ray absorption mechanisms relevant to this process.

4.3 Limitations and Future Work

While the present study provides valuable insights into the pair creation process, several limitations need to be addressed in future research:

1. **Effects of Temperature:** This study primarily focused on the influence of magnetic field strength and initial photon angle. However, temperature variations within the neutron star's magnetosphere may also play a crucial role in pair production probabilities. Future studies should investigate how temperature affects this process across different regions of the magnetosphere.

2. **Three-Dimensional Modeling:** The current model is based on a two-dimensional approach. Extending it to three dimensions could provide a more comprehensive understanding of photon trajectories and their interactions in the complex environment surrounding a neutron star.

3. **Quantum Electrodynamics (QED) Effects:** While our calculations consider key aspects of pair production, incorporating advanced QED effects—particularly in extremely strong magnetic fields—could further refine probability estimates. More sophisticated QED corrections should be included in future models to improve accuracy.

5 Conclusion

This study has explored the conditions necessary for one-photon pair creation in the strong magnetic fields near neutron stars. By analyzing the dependence of pair creation probability on photon energy, magnetic field strength, and initial emission angle, we have identified key factors influencing this process.

For magnetic field strengths ranging from 10^{12} Gauss to 10^{15} Gauss, the probability of pair creation increases exponentially. The results highlight that extremely strong magnetic fields create highly favorable conditions for electron-positron pair creation, with significant dependence on the initial photon angle.

In summary, the extreme magnetic environments of neutron stars play a crucial role in shaping photon interactions and radiation processes. This study enhances our understanding of pair creation near neutron stars and lays the groundwork for future research incorporating temperature effects, three-dimensional modeling, and advanced QED corrections to further refine theoretical predictions.

6 References

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